**Question** (1992 STEP II Q6)

Sketch the graphs of $y = \sec x$ and $y = \ln(2\sec x)$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. Show graphically that the equation

$$kx = \ln(2\sec x)$$

has no solution with $0 \leqslant x < \frac{1}{2}\pi$ if $k$ is a small positive number but two solutions if $k$ is large. Explain why there is a number $k_0$ such that

$$k_0 x = \ln(2\sec x)$$

has exactly one solution with $0 \leqslant x < \frac{1}{2}\pi$. Let $x_0$ be this solution, so that $0 \leqslant x_0 < \frac{1}{2}\pi$ and $k_0 x_0 = \ln(2\sec x_0)$. Show that

$$x_0 = \cot x_0 \ln(2\sec x_0).$$

Use any appropriate method to find $x_0$ correct to two decimal places. Hence find an approximate value for $k_0$.

**Question** (2004 STEP II Q7)

The function f is defined by

$$f(x) = 2\sin x - x\,.$$

Show graphically that the equation $f(x) = 0$ has exactly one root in the interval $[\frac{1}{2}\pi,\ \pi]$. This interval is denoted $I_0$. In order to determine the root, a sequence of intervals $I_1$, $I_2$, ... is generated in the following way. If the interval $I_n = [a_n, b_n]$, and $c_n = (a_n + b_n)/2$, then

$$I_{n+1} = \begin{cases} [a_n, c_n] & \text{if } \ f(a_n)f(c_n) < 0\,; \\ [c_n, b_n] & \text{if } \ f(c_n)f(b_n) < 0\,. \end{cases}$$

By using the approximations $\dfrac{1}{\sqrt{2}} \approx 0.7$ and $\pi \approx \sqrt{10}$, show that $I_2 = [\frac{1}{2}\pi,\ \frac{5}{8}\pi]$ and find $I_3$.