Question (1992 STEP II Q14)

xunit=1.0cm,yunit=1.0cm,algebraic=true,dimen=middle,dotstyle=o,dotsize=3pt 0,linewidth=0.5pt,arrowsize=3pt 2,arrowinset=0.25 (-3.36,-3.71)(5.32,4.49) [linewidth=0pt,linecolor=white,hatchcolor=black,fillstyle=hlines,hatchangle=45.0,hatchsep=0.19](-3,4.22)(-3,4)(5,4)(5,4.22) (-1,2)1 (3,2)1 (1,-1)1 (0,2)(0,-1) (2,2)(2,-1) (-2,2)(-2,-1) (4,2)(4,-1) -¿(-2,-1.44)(-2,-2) [tl](-2.25,-2.31)m_1g -¿(4,-1.44)(4,-2) [tl](3.74,-2.25)m_2g -¿(1,-1)(1.02,-2.78) [tl](0.72,-3.06)m_3g (-1,2)(-1,4) (3,2)(3,4) (-3,4)(5,4) [tl](-1.19,1.67)P_1 [tl](2.83,1.64)P_2 [tl](0.83,-0.5)P_3 [dotstyle=*](-1,2) [dotstyle=*](3,2) [dotstyle=*](1,-1) [dotstyle=*](-2,-1) [dotstyle=*](4,-1)

In the diagram P_1 and P_2 are smooth light pulleys fixed at the same height, and P_3 is a third smooth light pulley, freely suspended. A smooth light inextensible string runs over P_1, under P_3 and over P_2, as shown: the parts of the string not in contact with any pulley are vertical. A particle of mass m_3 is attached to P_3. There is a particle of mass m_1 attached to the end of the string below P_1 and a particle of mass m_2 attached to the other end, below P_2. The system is released from rest. Find the tension in the string, and show that the pulley P_3 will remain at rest if

$$4m_1m_2 = m_3(m_1 + m_2).$$

Question (1998 STEP I Q11)

Hank's Gold Mine has a very long vertical shaft of height l. A light chain of length l passes over a small smooth light fixed pulley at the top of the shaft. To one end of the chain is attached a bucket A of negligible mass and to the other a bucket B of mass m. The system is used to raise ore from the mine as follows. When bucket A is at the top it is filled with mass $2m$ of water and bucket B is filled with mass λm of ore, where $0 < \lambda < 1$. The buckets are then released, so that bucket A descends and bucket B ascends. When bucket B reaches the top both buckets are emptied and released, so that bucket B descends and bucket A ascends. The time to fill and empty the buckets is negligible. Find the time taken from the moment bucket A is released at the top until the first time it reaches the top again. This process goes on for a very long time. Show that, if the greatest amount of ore is to be raised in that time, then λ must satisfy the condition $\mathrm{f}'(\lambda) = 0$ where

$$\mathrm{f}(\lambda) = \frac{\lambda(1-\lambda)^{1/2}}{(1-\lambda)^{1/2} + (3+\lambda)^{1/2}}.$$

Question (2000 STEP II Q10)

A long light inextensible string passes over a fixed smooth light pulley. A particle of mass 4 kg is attached to one end A of this string and the other end is attached to a second smooth light pulley. A long light inextensible string BC passes over the second pulley and has a particle of mass 2 kg attached at B and a particle of mass of 1 kg attached at C. The system is held in equilibrium in a vertical plane. The string BC is then released from rest. Find the accelerations of the two moving particles. After T seconds, the end A is released so that all three particles are now moving in a vertical plane. Find the accelerations of A, B and C in this second phase of the motion. Find also, in terms of g and T, the speed of A when B has moved through a total distance of $0.6gT^2$ metres.

Question (2005 STEP II Q11)

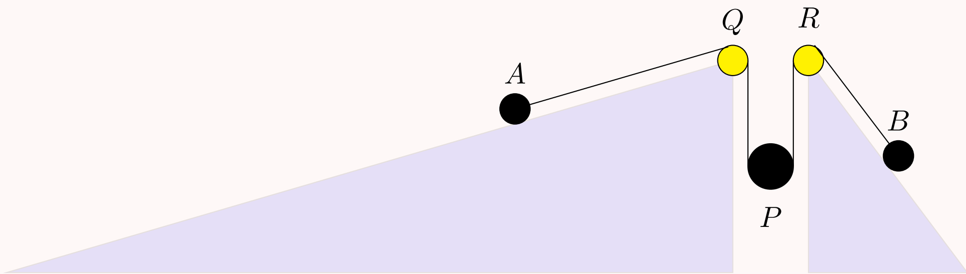
A plane is inclined at an angle $\arctan \frac{3}{4}$ to the horizontal and a small, smooth, light pulley P is fixed to the top of the plane. A string, APB, passes over the pulley. A particle of mass m_1 is attached to the string at A and rests on the inclined plane with AP parallel to a line of greatest slope in the plane. A particle of mass m_2, where $m_2 > m_1$, is attached to the string at B and hangs freely with BP vertical. The coefficient of friction between the particle at A and the plane is $\frac{1}{2}$.

The system is released from rest with the string taut. Show that the acceleration of the particles is $\dfrac{m_2 - m_1}{m_2 + m_1} g$.

At a time T after release, the string breaks. Given that the particle at A does not reach the pulley at any point in its motion, find an expression in terms of T for the time after release at which the particle at A reaches its maximum height. It is found that, regardless of when the string broke, this time is equal to the time taken by the particle at A to descend from its point of maximum height to the point at which it was released. Find the ratio $m_1 : m_2$. [Note that $\arctan \frac{3}{4}$ is another notation for $\tan^{-1} \frac{3}{4}$.]

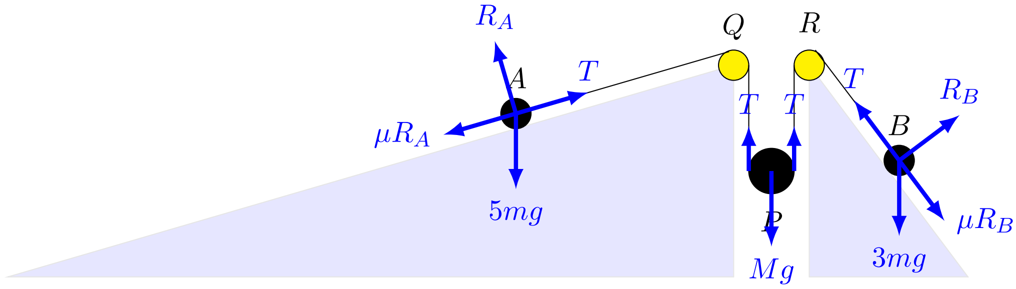
Question (2012 STEP I Q11)

The diagram shows two particles, A of mass $5m$ and B of mass $3m$, connected by a light inextensible string which passes over two smooth, light, fixed pulleys, Q and R, and under a smooth pulley P which has mass M and is free to move vertically. Particles A and B lie on fixed rough planes inclined to the horizontal at angles of $\arctan\frac{7}{24}$ and $\arctan\frac{4}{3}$ respectively. The segments AQ and RB of the string are parallel to their respective planes, and segments QP and PR are vertical. The coefficient of friction between each particle and its plane is μ.



(i) Given that the system is in equilibrium, with both A and B on the point of moving up their planes, determine the value of μ and show that $M = 6m$.

(ii) In the case when $M = 9m$, determine the initial accelerations of A, B and P in terms of g.



First note our triangles are 7-24-25 and 3-4-5 triangles, so we can easily calculate sin and cos of our angles.

(i)

$$\text{N2}(\uparrow, P): \quad 2T - Mg = 0$$

$$\text{N2}(\perp AQ): \quad R_A - 5mg \cdot \frac{24}{25} = 0$$

$$\Rightarrow \quad R_A = \frac{24}{5}mg$$

$$\text{N2}(\parallel AQ): \quad T - \mu R_A - 5mg \cdot \frac{7}{25} = 0$$

$$\Rightarrow \quad T = \frac{1}{5}mg\,(7 + 24\mu)$$

$$\text{N2}(\perp BR): \quad R_B - 3mg \cdot \frac{3}{5} = 0$$

$$\Rightarrow \qquad R_B = \frac{9}{5}mg$$

$$\text{N2}(\parallel AQ): \qquad T - \mu R_B - 3mg \cdot \frac{4}{5} = 0$$

$$\Rightarrow \qquad T = \frac{1}{5}mg\,(12 + 9\mu)$$

$$\Rightarrow \qquad 12 + 9\mu = 7 + 24\mu$$

$$\Rightarrow \qquad \mu = \frac{5}{15} = \frac{1}{3}$$

$$\Rightarrow \qquad Mg = 2 \cdot \frac{1}{5} \cdot mg \cdot (7 + 24 \cdot \frac{1}{3})$$

$$= 6mg$$

$$\Rightarrow \qquad M = 6m$$

(ii) Assuming $\mu = \frac{1}{3}$

Question (2014 STEP I Q11)

The diagrams below show two separate systems of particles, strings and pulleys. In both systems, the pulleys are smooth and light, the strings are light and inextensible, the particles move vertically and the pulleys labelled with P are fixed. The masses of the particles are as indicated on the diagrams.

xunit=0.7cm,yunit=0.7cm,algebraic=true,dimen=middle,dotstyle=o,dotsize=3pt 0,linewidth=0.3pt,arrowsize=3pt 2,arrowinset=0.25 (-0.6,-5.31)(12.33,7.94) (1,-3)(1,-2)(0,-2)(0,-3) (4.01,-1)(4.01,0)(3.01,0)(3.01,-1) (10.3,-1)(10.3,0)(9.3,0)(9.3,-1) (11.76,-3)(11.76,-2)(10.76,-2)(10.76,-3) (8,0)(8,1)(7,1)(7,0) (2,6)1.05 (9,6)1.05 (3.5,6)(3.49,0) (0.5,6.01)(0.5,-2) (1,-3)(1,-2) (1,-2)(0,-2) (0,-2)(0,-3) (0,-3)(1,-3) (4.01,-1)(4.01,0) (4.01,0)(3.01,0) (3.01,0)(3.01,-1) (3.01,-1)(4.01,-1) (7.5,5.99)(7.51,1.01) (10.5,6)(10.48,3.98) (10.5,3.25)0.51 (9.77,3.26)(9.77,-0.01) (11.23,3.26)(11.23,-2) (10.3,-1)(10.3,0) (10.3,0)(9.3,0) (9.3,0)(9.3,-1) (9.3,-1)(10.3,-1) (11.76,-3)(11.76,-2) (11.76,-2)(10.76,-2) (10.76,-2)(10.76,-3) (10.76,-3)(11.76,-3) (8,0)(8,1) (8,1)(7,1) (7,1)(7,0) (7,0)(8,0) [tl](0.16,-2.33)M [tl](3.2,-0.4)m [tl](7.18,0.67)M [tl](9.46,-0.4)m_1 [tl](10.91,-2.4)m_2 [tl](1.82,6.14)P [tl](8.84,6.14)P [tl](10.22,3.48)P_1 [tl](1.06,-3.72)System I [tl](8.11,-3.68)System II

(i) For system I show that the acceleration, a_1, of the particle of mass M, measured in the downwards direction, is given by

$$a_1 = \frac{M-m}{M+m}\,g\,,$$

where g is the acceleration due to gravity. Give an expression for the force on the pulley due to the tension in the string.

(ii) For system II show that the acceleration, a_2, of the particle of mass M, measured in the downwards direction, is given by

$$a_2 = \frac{M-4\mu}{M+4\mu}\,g\,,$$

where $\mu = \dfrac{m_1 m_2}{m_1+m_2}$. In the case $m = m_1 + m_2$, show that $a_1 = a_2$ if and only if $m_1 = m_2$.