Question (2000 STEP I Q11)

A rod AB of length 0.81 m and mass 5 kg is in equilibrium with the end A on a rough floor and the end B against a very rough vertical wall. The rod is in a vertical plane perpendicular to the wall and is inclined at 45° to the horizontal. The centre of gravity of the rod is at G, where $AG = 0.21$ m. The coefficient of friction between the rod and the floor is 0.2, and the coefficient of friction between the rod and the wall is 1.0. Show that the friction cannot be limiting at both A and B.

A mass of 5 kg is attached to the rod at the point P such that now the friction is limiting at both A and B. Determine the length of AP.

Question (2003 STEP II Q10)

A bead B of mass m can slide along a rough horizontal wire. A light inextensible string of length 2ℓ has one end attached to a fixed point A of the wire and the other to B. A particle P of mass $3m$ is attached to the mid-point of the string and B is held at a distance ℓ from A. The bead is released from rest.

Let a_1 and a_2 be the magnitudes of the horizontal and vertical components of the initial acceleration of P. Show by considering the motion of P relative to A, or otherwise, that $a_1 = \sqrt{3}a_2$. Show also that the magnitude of the initial acceleration of B is $2a_1$. Given that the frictional force opposing the motion of B is equal to $(\sqrt{3}/6)R$, where R is the normal reaction between B and the wire, show that the magnitude of the initial acceleration of P is $g/18$.

Question (2005 STEP II Q9)

Two particles, A and B, of masses m and $2m$, respectively, are placed on a line of greatest slope, ℓ, of a rough inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between A and the plane is $\frac{1}{6}\sqrt{3}$ and the coefficient of friction between B and the plane is $\frac{1}{3}\sqrt{3}$. The particles are at rest with B higher up ℓ than A and are connected by a light inextensible string which is taut. A force P is applied to B.

(i) Show that the least magnitude of P for which the two particles move upwards along ℓ is $\frac{11}{8}\sqrt{3}\,mg$ and give, in this case, the direction in which P acts.

(ii) Find the least magnitude of P for which the particles do not slip downwards along ℓ.

Question (2007 STEP I Q9)

A particle of weight W is placed on a rough plane inclined at an angle of θ to the horizontal. The coefficient of friction between the particle and the plane is μ. A horizontal force X acting on the particle is just sufficient to prevent the particle from sliding down the plane; when a horizontal force kX acts on the particle, the particle is about to slide up the plane. Both horizontal forces act in the vertical plane containing the line of greatest slope. Prove that

$$(k-1)\left(1+\mu^2\right)\sin\theta\cos\theta = \mu\,(k+1)$$

and hence that $k \geq \dfrac{(1+\mu)^2}{(1-\mu)^2}$.

Question (2008 STEP I Q11)

A straight uniform rod has mass m. Its ends P_1 and P_2 are attached to small light rings that are constrained to move on a rough rigid circular wire with centre O fixed in a vertical plane, and the angle P_1OP_2 is a right angle. The rod rests with P_1 lower than P_2, and with both ends lower than O. The coefficient of friction between each of the rings and the wire is μ. Given that the rod is in limiting equilibrium (i.e. on the point of slipping at both ends), show that

$$\tan\alpha = \frac{1-2\mu-\mu^2}{1+2\mu-\mu^2}\,,$$

where α is the angle between P_1O and the vertical ($0 < \alpha < 45°$). Let θ be the acute angle between the rod and the horizontal. Show that $\theta = 2\lambda$, where λ is defined by $\tan\lambda = \mu$ and $0 < \lambda < 22.5°$.

Question (2008 STEP II Q11)

A wedge of mass km has the shape (in cross-section) of a right-angled triangle. It stands on a smooth horizontal surface with one face vertical. The inclined face makes an angle θ with the horizontal surface. A particle P, of mass m, is placed on the inclined face and released from rest. The horizontal face of the wedge is smooth, but the inclined face is rough and the coefficient of friction between P and this face is μ.

(i) When P is released, it slides down the inclined plane at an acceleration a relative to the wedge. Show that the acceleration of the wedge is

$$\frac{a\cos\theta}{k+1}\,.$$

To a stationary observer, P appears to descend along a straight line inclined at an angle $45°$ to the horizontal. Show that

$$\tan\theta = \frac{k}{k+1}\,.$$

In the case $k = 3$, find an expression for a in terms of g and μ.

(ii) What happens when P is released if $\tan\theta \le \mu$?

Question (2012 STEP II Q10)

A hollow circular cylinder of internal radius r is held fixed with its axis horizontal. A uniform rod of length $2a$ (where $a < r$) rests in equilibrium inside the cylinder inclined at an angle of θ to the horizontal, where $\theta \neq 0$. The vertical plane containing the rod is perpendicular to the axis of the cylinder. The coefficient of friction between the cylinder and each end of the rod is μ, where $\mu > 0$. Show that, if the rod is on the point of slipping, then the normal reactions R_1 and R_2 of the lower and higher ends of the rod, respectively, on the cylinder are related by

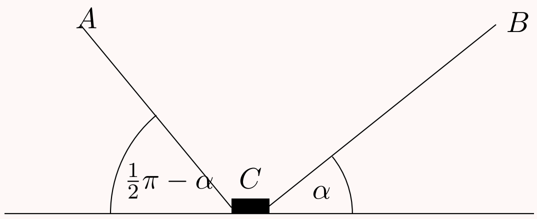
$$\mu(R_1 + R_2) = (R_1 - R_2)\tan\phi$$

where ϕ is the angle between the rod and the radius to an end of the rod. Show further that

$$\tan\theta = \frac{\mu r^2}{r^2 - a^2(1+\mu^2)}\,.$$

Deduce that $\lambda < \phi$, where $\tan\lambda = \mu$.

Question (2013 STEP I Q11)



The diagram shows a small block C of weight W initially at rest on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. Two light strings, AC and BC, are attached to the block, making angles $\frac{1}{2}\pi - \alpha$ and α to the horizontal, respectively. The tensions in AC and BC are $T\sin\beta$ and $T\cos\beta$ respectively, where $0 < \alpha + \beta < \frac{1}{2}\pi$.

(i) In the case $W > T\sin(\alpha+\beta)$, show that the block will remain at rest provided

$$W\sin\lambda \geq T\cos(\alpha+\beta-\lambda)\,,$$

where λ is the acute angle such that $\tan\lambda = \mu$.

(ii) In the case $W = T\tan\phi$, where $2\phi = \alpha+\beta$, show that the block will start to move in a direction that makes an angle ϕ with the horizontal.

Question (2016 STEP I Q9)

A horizontal rail is fixed parallel to a vertical wall and at a distance d from the wall. A uniform rod AB of length $2a$ rests in equilibrium on the rail with the end A in contact with the wall. The rod lies in a vertical plane perpendicular to the wall. It is inclined at an angle θ to the vertical (where $0 < \theta < \frac{1}{2}\pi$) and $a \sin\theta < d$, as shown in the diagram.

wwwwww0.4 0.4 0.4 xunit=1.0cm,yunit=1.0cm (-2.02,-0.1)(6.98,4.98)
xunit=1.0cm,yunit=1.0cm,algebraic=true,dotstyle=o,dotsize=3pt
0,linewidth=0.8pt,arrowsize=3pt 2,arrowinset=0.25
[linewidth=6.2pt,linestyle=dashed,dash=11pt
1pt,linecolor=wwwwww](-0.12,5)(-0.12,0)
[linewidth=2pt](0,1.86)(5.86,3.34) [linestyle=dotted](4,4.48)(4,0.58) [tl](0.15,2.4)θ
[linestyle=dotted](0,4)(3.98,4) [tl](1.86,4.40)d [tl](0.12,1.78)A [tl](5.66,3.14)B
0.24738636312925451.57081*0.80*cos(t)+0*0.80*sin(t)+0—0*0.80*cos(t)+1*0.80*sin(t)+1.86
[dotsize=5pt 0,dotstyle=*](4,2.78)

The coefficient of friction between the rod and the wall is μ, and the coefficient of friction between the rod and the rail is λ.

Show that in limiting equilibrium, with the rod on the point of slipping at both the wall and the rail, the angle θ satisfies

$$d^2\theta = a\big((\lambda + \mu)\cos\theta + (1 - \lambda\mu)\sin\theta\big)\,.$$

Derive the corresponding result if, instead, $a\sin\theta > d$.